

Centre vortices and their friends

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We report two probes of centre vortices in pure SU(2). First, we attempt to generate plaquette-size Z(2) vortices in a quasi-random way to compare the structure with those of projected centre vortices from the full theory; our prescription is, however, not completely random. Second, we test the effect of centre vortices on Wilson loops as a function of position. This shows clearly that the major effect is the piercing of the Wilson loop regardless of the position of the vortex inside.

1. INTRODUCTION

One approach to QCD confinement is that of centre vortices, where the key degrees of freedom are those in the centre of the gauge group, $Z(N)$ in the case of SU(N). Numerical results for the case of SU(2) (also treated here) show that this is fruitful. The main technique is that of centre projection [2]: fixing to a gauge where the links are as close as possible to a centre element, then projecting to that element, leaving a lattice of Z(2) links; negative plaquettes are called P-vortices and are interpreted as the source of confinement. Here we examine two related issues.

2. RANDOM VORTICES

A random gas of infinitely long vortices will cause linear confinement. This is too simplistic, but maybe can teach us something: indeed it gives about the right string tension from measured vortex densities. Viewed in four dimensions the vortices are defined by closed surfaces; confinement survives only so long as this surface percolates throughout the space-time manifold, and hence deconfinement may be due to loss of percolation [1].

This has all been argued from the point of view of taking SU(2) and reducing the degrees of freedom to the bare essentials. Here we shall attempt the opposite: to construct an (approximately) random vortex picture. Truly random vortices are difficult because of the strong coupling of adjacent plaquettes via the links, even with no gauge

coupling present. Our lattice and observables are as in the projected Z(2) theory. We use the following procedure:

- Create a random set of links, either ± 1 with 50% probability ('random start') or set to unity ('frozen start').
- Let v = density of negative plaquettes (corresponding to vortices); initially $v \sim 0.5$ or $v \equiv 0$. Pick a target $v = v_T$ chosen to correspond to the mean density of P-vortices in SU(2). At $\beta = 2.3$, $v_T \approx 0.0945$; at $\beta = 2.4$, $v_T \approx 0.0602$.
- Pick a link at random in the lattice. Flip the sign of this link either (i) if it does not alter v or (ii) if it alters v towards v_T .
- Continue until v_T is achieved. Because of condition (i) it is useful to attempt to flip links already considered. In the case of the frozen start, we have tried further to make the vortices independent by making sets of flips which do not affect the overall vortex density.
- Generate many configurations of this sort and analyse them as a Monte Carlo sample. Note that here there is no Markov process, and hence no fluctuating action; in a sense our ensemble is microcanonical.

There is a bias in this procedure because we flip links attached to sets of plaquettes predominantly of one sign, hence our vortices are not

	$\beta = 2.3$	Random	Frozen
(a)	0.0945(6)	0.0945	0.0945
(b)	0.0727(9)	0.01060(11)	0.9876(2)
(c)	10890(60)	11631(1)	146(2)
(d)	0.1362(2)	0.189	0.189
(e)	0.145(7)	0.41(3)	0
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	$\beta = 2.4$		
(a)	0.06015(6)	0.0602	0.0602
(b)	0.1125(12)	0.00494(6)	0.99742(3)
(c)	20600(100)	23553(1)	61.0(6)
(d)	0.0708(11)	0.12	0.12
(e)	0.079(7)	0.164(3)	0

Table 1

Comparison between SU(2) and quasi-random vortices. The columns show SU(2), and quasi-random vortices starting from random or frozen links; the rows are (a) vortex density v (fixed for the quasi-random vortices), (b) fraction of P-plaquettes *not* in the largest cluster, (c) the number of P-plaquettes in the largest cluster, (d) the string tension based on large scale simulations of full SU(2) [3] or purely random vortices, and (e) that actually measured from vortices, see text.

truly random. We could instead have chosen the target v to correspond to the SU(2) string tension on the assumption of truly random vortices. Our actual choice reflects a desire to look at the cluster properties of vortices.

2.1. Results

Fig. 1 shows results on bulk lattices, 12^4 for $\beta = 2.3$ and 16^4 for $\beta = 2.4$. The string tension is shown both for the two ideal cases (from a large scale run in full SU(2) and for fully random vortices) and as measured from vortices. In the quasi-random case with the random start, Creutz ratios show a string tension which for small loops lies near the expected value ($2v$) but which increases for larger loops. The results shown are from a full potential calculation where this increase tends to level out, although with some curvature, giving a rather larger string tension; the form fit to is necessarily somewhat ad hoc and here we have included a quadratic part. Furthermore, in the frozen start the vortices lack confinement and hence show in effect a repulsion.

These are sizeable effects; a more truly random method will be needed for a more realistic comparison. An effective action would also presumably help [5].

Nonetheless, we examine cluster properties by methods similar to ref. [4], dividing vortices into two clusters where the surfaces touch only along an edge. This difference between touching and joining is a lattice effect which makes a noticeable impact — almost tripling the number of vortices not in the largest (percolating) cluster for the case of SU(2) with $\beta = 2.3$ with the random start, and increasing the largest cluster size dramatically for the frozen start. Of course we would prefer to detect vortices directly with their physical size.

2.2. The deconfining transition

We have also examined a lattice in the deconfined phase, using Polyakov loops L as the order parameter, although it is maybe unlikely that homogeneous random vortices alone can be sufficient to explain deconfinement. The lattice results show that $\langle |L| \rangle$ goes to 1 for small vortex density, but this is expected simply due to the fact that neighbouring loops are effectively Wilson loops with an area equal to the finite temperature extent of the lattice, and hence correlated by the vanishing string tension. There is no sign of a phase transition, nor finite size scaling behaviour. It may well be important to have the vortex surface orientated predominantly parallel to, and hence not piercing, temporal Wilson loops; it is not clear such an effect can come from just the Z(2) degrees of freedom.

3. PROBING WILSON LOOPS

The plaquette-sized P-vortices are expected to have a topological effect on Wilson loops, depending only on whether a vortex pierces the loop. We investigate this by looking at the correlations between P-vortices and Wilson loops.

Our method is the following (fig. 1). We take a plaquette P on the centre-projected lattice within a wilson loop W , a certain distance from the centre of the loop. For present purposes we shall simply take the distance r to be the number of plaquettes diagonally from the centre of the loop,

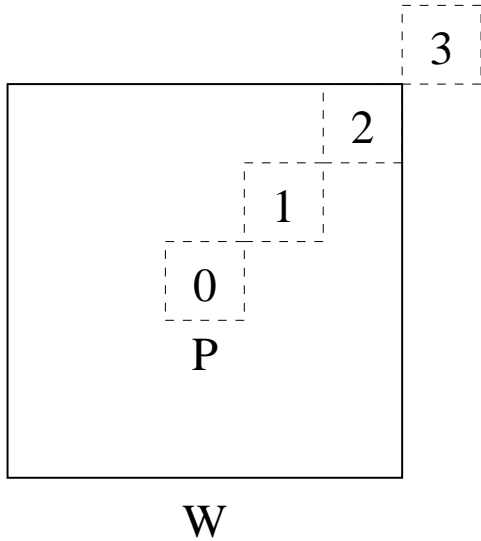


Figure 1. Placement of a plaquette-sized probe in a Wilson loop, showing the diagonal distance r ; for a loop size L with L odd, $r = (L + 1)/2$ is just outside the loop.

as in the diagram. If $P = 1$, we ignore W and pass on to the next one; if $P = -1$ we examine the value of W . After sampling over many configurations, we can form an average $\langle W_{P=-1}(r) \rangle$. Note that in examining P we take no account whatsoever of other centre plaquettes inside (or outside) W ; the effect is purely the correlation between the Wilson loop and a centre vortex at the given position, whether or not the loop is pierced by other vortices. To achieve sufficiently large correlations we are restricted to loops of sizes that have $\mathcal{O}(1)$ vortices inside. Clearly, if there is no correlation, $\langle W_{P=-1}(r) \rangle = \langle W \rangle$. As a control, we have performed the same experiment replacing P with the sign of a gauge plaquette G located in the same place.

The results (fig. 2) show that $\langle W_{P=-1}(r) \rangle$ is rather flat inside the loop, but with a significant correlation. In contrast, the values of $\langle W_{G=-1}(r) \rangle$ vary much more widely over the inside of the loop. This is a sign that the dominant effect of the vortex is given by whether or not it pierces the loop, regardless of where it does

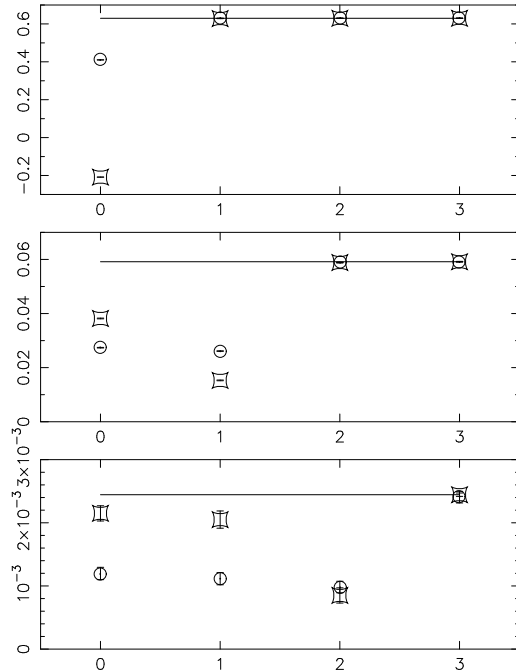


Figure 2. Probes $W_{P=-1}$ (circles) and $W_{G=-1}$ (squashy squares) for $\beta = 2.4$, 16^4 lattice, for (top) 1×1 , 3×3 , 5×5 (bottom) loops.

so, an effect not expected and not shown by the sign of the full gauge plaquette. Both probes become uncorrelated very quickly when outside the loops. For gauge plaquettes this can be understood from strong coupling; such plaquettes only appear in quite high order. For P-plaquettes the natural interpretation is that vortices not piercing the Wilson loop have no effect on it. However, if the vortices really correspond to extended physical objects, it is not clear why the change from inside to outside should be so sharp; this raises questions about the size of the vortex core.

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